Quantum Field Theory: Foundations and Selected Recent Directions

A short notes-style document with equation-heavy exposition

February 22, 2026

Abstract

These notes collect a self-contained set of core quantum field theory (QFT) definitions and derivations (path integrals, generating functionals, perturbation theory, renormalization, gauge fields) together with brief pointers to a few active areas (amplitudes, conformal bootstrap, effective field theory, and applications to condensed matter). The emphasis is on formulas and standard manipulations.

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1 Conventions and kinematics

We work in $d = 4$ Minkowski signature $\eta_{\mu\nu} = \mathrm{diag}(+,-,-,-)$ unless stated otherwise. The action is

$$S[\phi] = \int d^4x\, \mathcal{L}(\phi, \partial\phi), \qquad \exp(iS) \text{ in the path integral.}$$

Fourier transform conventions:

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\, \tilde{\phi}(p), \qquad p \cdot x = p_\mu x^\mu.$$

2 The free scalar field

2.1 Classical theory

The Klein–Gordon Lagrangian is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}m^2\phi^2.$$

The Euler–Lagrange equation gives

$$(\Box + m^2)\phi = 0, \qquad \Box = \partial_\mu\partial^\mu.$$

The canonical momentum and Hamiltonian density are

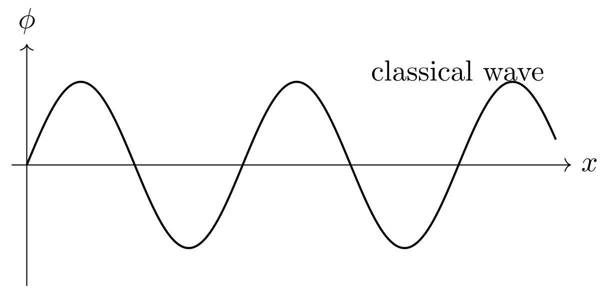
$$\pi(x) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)} = \dot{\phi}(x), \qquad \mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2.$$

2.2 A pictorial intuition (waves, modes, quanta)

The classical field $\phi(t, \mathbf{x})$ behaves like a continuum of coupled oscillators. In Fourier space each momentum mode evolves independently:

$$\tilde{\phi}(t, \mathbf{p}) \text{ solves } \ddot{\tilde{\phi}}(t, \mathbf{p}) + \omega_{\mathbf{p}}^2 \tilde{\phi}(t, \mathbf{p}) = 0, \qquad \omega_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}.$$

Quantization turns each mode into a harmonic oscillator with ladder operators and discrete excitations ("particles").



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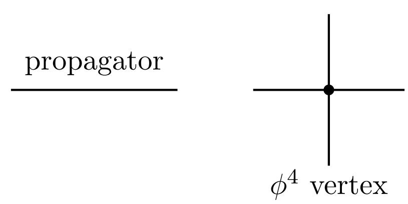
2.4 Diagrammatic language (Wick contractions)

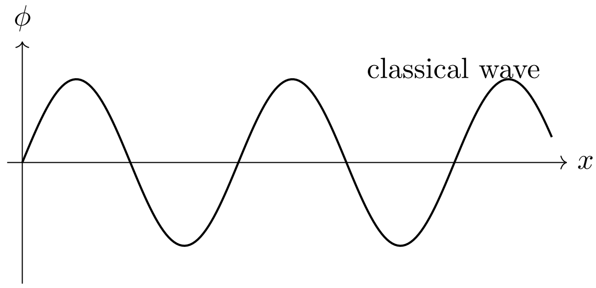
In perturbation theory, terms in the expansion are efficiently organized by Feynman diagrams. A propagator Δ_F is drawn as a line; a ϕ^4 interaction is a 4-valent vertex.



For example, the connected two-point function in ϕ^4 receives a one-loop correction proportional to

$$\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon},$$

which is divergent and motivates renormalization. Quantization turns each mode into a harmonic oscillator with ladder operators and discrete excitations ("particles").



2.5 Quantization and propagator

Canonical quantization imposes

$$[\phi(t,\mathbf{x}),\pi(t,\mathbf{y})] = i\delta^{(3)}(\mathbf{x}-\mathbf{y}), \qquad [\phi,\phi]=[\pi,\pi]=0.$$

2.6 A geometric picture (connections and curvature)

Mathematically, A_μ is a connection one-form and $F_{\mu\nu}$ is its curvature. Gauge transformations act as

$$A_\mu \mapsto U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}, \qquad F_{\mu\nu} \mapsto U F_{\mu\nu} U^{-1},$$

so $\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ is gauge-invariant.

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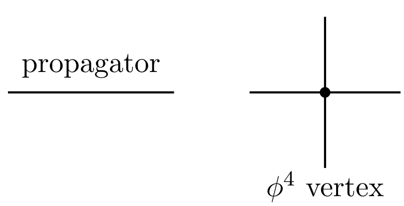

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$$\Delta_F(x-y) = \langle 0|T\phi(x)\phi(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\,\frac{i\,e^{-ip\cdot(x-y)}}{p^2-m^2+i\epsilon}.$$

3 Path integrals and generating functionals

3.1 Gaussian functional integrals

For a real scalar field with source J,

$$Z[J] = \int \mathcal{D}\phi \, \exp\left\{ i \int d^4x \left[\frac{1}{2}\phi(x)(\Box + m^2 - i\epsilon)\phi(x) + J(x)\phi(x) \right] \right\}.$$

Completing the square yields

$$Z[J] = Z[0] \exp\left\{ -\frac{i}{2} \int d^4x \, d^4y \; J(x)\Delta_F(x-y)J(y) \right\}.$$

Correlation functions follow from

$$\langle 0|T\phi(x_1)\cdots\phi(x_n)|0\rangle = \frac{1}{i^n} \frac{\delta^n}{\delta J(x_1)\cdots\delta J(x_n)} \frac{Z[J]}{Z[0]}\bigg|_{J=0}.$$

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so $\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ is gauge-invariant.

3.3 Interacting theory and perturbation

For ϕ^4 theory,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4, \qquad Z[J] = \int \mathcal{D}\phi \, e^{iS[\phi] + i\int J\phi}.$$

Split $S = S_0 + S_I$ and write

$$Z[J] = \exp\left(-i \int d^4x \, \frac{\lambda}{4!} \left(\frac{1}{i} \frac{\delta}{\delta J(x)} \right)^4 \right) Z_0[J],$$

which is the compact form of Wick expansion and Feynman rules.

4 Renormalization: one-loop landmarks

In dimensional regularization with $d = 4 - 2\epsilon$ and scale μ, define bare and renormalized parameters (schematically)

$$\phi_0 = Z_\phi^{1/2}\phi, \qquad m_0^2 = m^2 + \delta m^2, \qquad \lambda_0 = \mu^{2\epsilon}(\lambda + \delta\lambda).$$

The one-loop β-function in ϕ^4 theory in four dimensions is

$$\beta(\lambda) = \mu\frac{d\lambda}{d\mu} = \frac{3}{16\pi^2}\lambda^2 + \mathcal{O}(\lambda^3),$$

and the Callan–Symanzik equation for renormalized n-point functions $G^{(n)}$ reads

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + n\gamma_\phi(\lambda) - \gamma_m(\lambda)m^2\frac{\partial}{\partial m^2}\right)G^{(n)} = 0.$$

5 Gauge fields (sketch)

For a non-abelian gauge field $A_\mu = A_\mu^a T^a$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \qquad \mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}).$$

Gauge fixing and ghosts (in covariant gauges) add

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\xi}(\partial_\mu A_a^\mu)^2, \qquad \mathcal{L}_{\mathrm{gh}} = \partial_\mu \bar{c}^a (D^\mu c)^a.$$

6 Selected recent directions (brief, non-exhaustive)

6.1 Scattering amplitudes

Modern amplitude methods often exploit on-shell constraints. For example, color-ordered tree amplitudes in gauge theory can be built via BCFW recursion from complex momentum shifts that preserve $p_i^2 = 0$ and overall momentum conservation.

6.2 A "dictionary" figure: core objects in QFT

Picture / object	Symbol	What it computes
Line	$\Delta_F(x-y)$	propagation / correlations
Vertex	$-i\lambda$ or $-ig\gamma^\mu$	interactions
Loop	$\int d^dk$	quantum fluctuations / UV behavior
Generating functional	$Z[J]$	all correlators via functional derivatives
Effective action	$\Gamma[\phi]$	1PI physics, quantum corrected EOM

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Generating functional	$Z[J]$	all correlators via functional derivatives
Effective action	$\Gamma[\phi]$	1PI physics, quantum corrected EOM

6.4 Conformal field theory and bootstrap

In a CFT, the two-point function of a scalar primary $\mathcal{O}$ of scaling dimension Δ is fixed by symmetry:

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{C_{\mathcal{O}}}{(x^2)^\Delta}.$$

The four-point function admits an OPE expansion whose crossing symmetry constraints become powerful numerical equations (the bootstrap program).

6.5 Effective field theory (EFT)

At energies $E \ll \Lambda$, integrate out heavy modes to obtain

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{light}} + \sum_k \frac{c_k}{\Lambda^{k-4}}\mathcal{O}_k,$$

with power counting organizing predictions and uncertainties.

7 Further reading (starter list)

Standard textbooks: Peskin–Schroeder; Weinberg (vols. I–III); Srednicki; Schwartz. For modern amplitudes: Elvang–Huang. For CFT/bootstrap: Poland–Rychkov–Vichi (review). For EFT: Georgi (classic) and many modern reviews.